What Is Claimed Is:

1.   1.   A method for using a computer system to solve a global

2. optimization problem specified by a function f and a set of equality constraints,

3. the method comprising:

4.   receiving a representation of the function f and the set of equality

5. constraints $q_i(\mathbf{x}) = 0$ $(i=1,...,r)$ at the computer system, wherein f is a scalar

6. function of a vector $\mathbf{x} = (x_1, x_2, x_3, ... x_n)$;

7.   storing the representation in a memory within the computer system;

8.   performing an interval equality constrained global optimization process to

9. compute guaranteed bounds on a globally minimum value of the function $f(\mathbf{x})$

10. subject to the set of equality constraints;

11.   wherein performing the interval equality constrained global optimization

12. process involves,

13.   applying term consistency to a set of relations associated

14. with the interval equality constrained global optimization problem

15. over a subbox $\mathbf{X}$, and excluding any portion of the subbox $\mathbf{X}$ that

16. violates any of these relations,

17.   applying box consistency to the set of relations associated

18. with the interval equality constrained global optimization problem

19. over the subbox $\mathbf{X}$, and excluding any portion of the subbox $\mathbf{X}$ that

20. violates any of the relations, and

21.   performing an interval Newton step for the interval

22. equality constrained global optimization problem over the subbox

23. $\mathbf{X}$.

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1   2.   The method of claim 1, wherein applying term consistency to the

2 set of relations involves applying term consistency to the set of equality

3 constraints $q_i(\mathbf{x}) = 0$ $(i=1,...,r)$ over the subbox $\mathbf{X}$.

1   3.   The method of claim 1, wherein applying box consistency to the

2 set of relations involves applying box consistency to the set of equality constraints

3 $q_i(\mathbf{x}) = 0$ $(i=1,...,r)$ over the subbox $\mathbf{X}$.

1   4.   The method of claim 1,

2   wherein performing the interval equality constrained global optimization

3 process involves,

4   keeping track of a least upper bound f_bar of the function

5          $f(\mathbf{x})$, and

6   removing from consideration any subbox for which

7          $inf(f(\mathbf{X})) > f\_bar$;

8   wherein applying term consistency to the set of relations involves applying

9 term consistency to the f_bar inequality $f(\mathbf{x}) \leq f\_bar$ over the subbox $\mathbf{X}$.

1   5.   The method of claim 4, wherein applying box consistency to the

2 set of relations involves applying box consistency to the f_bar inequality

3 $f(\mathbf{x}) \leq f\_bar$ over the subbox $\mathbf{X}$.

1   6.   The method of claim 1,

2 wherein performing the interval equality constrained global optimization process

3 involves preconditioning the set of equality constraints through multiplication by

4 an approximate inverse matrix $\mathbf{B}$ to produce a set of preconditioned equality

5 constraints; and

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6   wherein applying term consistency to the set of relations involves applying

7   term consistency to the set of preconditioned equality constraints over the subbox

8   **X**.

1   7.   The method of claim 6, wherein applying box consistency to the

2   set of relations involves applying box consistency to the set of preconditioned

3   equality constraints over the subbox **X**.

1   8.   The method of claim 1, wherein performing the interval Newton

2   step involves performing the interval Newton step on the John conditions.

1   9.   The method of claim 1, wherein prior to performing the interval

2   Newton step on the John conditions, the method further comprises performing a

3   linearization test to determine whether to perform the Newton step on the John

4   conditions.

1   10.   The method of claim 1, wherein performing the interval equality

2   constrained global optimization process involves:

3   evaluating a first termination condition;

4   wherein the first termination condition is TRUE if the width of the subbox

5   **X** is less than a pre-specified value, ε_X, and the width of $f(\mathbf{X})$ is less than a pre-

6   specified value, ε_F; and

7   if the first termination condition is TRUE, terminating further splitting of

8   the subbox **X**.

1   11.   The method of claim 1, wherein performing the interval Newton

2   step involves:

1   computing $J(x,X)$, wherein $J(x,X)$ is the Jacobian of the function $\mathbf{f}$

2   evaluated as a function of x over the subbox $\mathbf{X}$; and

3   determining if $J(x,X)$ is regular as a byproduct of solving for the subbox $\mathbf{Y}$

4   that contains values of y that satisfy $M(x,X)(y-x) = r(x)$, where

5    $M(x,X) = BJ(x,X)$, $r(x) = -Bf(x)$, and $\mathbf{B}$ is an approximate inverse of the center of

6    $J(x,X)$.

1   12.   A computer-readable storage medium storing instructions that

2   when executed by a computer cause the computer to perform a method for using a

3   computer system to solve a global optimization problem specified by a function f

4   and a set of equality constraints, the method comprising:

5   receiving a representation of the function f and the set of equality

6   constraints $q_i(x) = 0$ ($i=1,...,r$) at the computer system, wherein f is a scalar

7   function of a vector $\mathbf{x} = (x_1, x_2, x_3, ... x_n)$;

8   storing the representation in a memory within the computer system;

9   performing an interval equality constrained global optimization process to

10   compute guaranteed bounds on a globally minimum value of the function $f(x)$

11   subject to the set of equality constraints;

12   wherein performing the interval equality constrained global optimization

13   process involves,

14   applying term consistency to a set of relations associated

15   with the interval equality constrained global optimization problem

16   over a subbox $\mathbf{X}$, and excluding any portion of the subbox $\mathbf{X}$ that

17   violates any of these relations,

18   applying box consistency to the set of relations associated

19   with the interval equality constrained global optimization problem

20   over the subbox **X**, and excluding any portion of the subbox **X** that

21   violates any of the relations, and

22   performing an interval Newton step for the interval

23   equality constrained global optimization problem over the subbox

24   **X**.

1   13.   The computer-readable storage medium of claim 12, wherein

2   applying term consistency to the set of relations involves applying term

3   consistency to the set of equality constraints $q_i(\mathbf{x}) = 0$ ($i=1,...,r$) over the subbox

4   **X**.

1   14.   The computer-readable storage medium of claim 12, wherein

2   applying box consistency to the set of relations involves applying box consistency

3   to the set of equality constraints $q_i(\mathbf{x}) = 0$ ($i=1,...,r$) over the subbox **X**.

1   15.   The computer-readable storage medium of claim 12,

2   wherein performing the interval equality constrained global optimization

3   process involves,

4   keeping track of a least upper bound f_bar of the function

5    $f(\mathbf{x})$, and

6   removing from consideration any subbox for which

7    $inf(f(\mathbf{X})) > f\_bar$;

8   wherein applying term consistency to the set of relations involves applying

9   term consistency to the f_bar inequality $f(\mathbf{x}) \le f\_bar$ over the subbox **X**.

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1   16.   The computer-readable storage medium of claim 15, wherein
2   applying box consistency to the set of relations involves applying box consistency
3   to the *f_bar* inequality $f(\mathbf{x}) \leq f\_bar$ over the subbox **X**.

1   17.   The computer-readable storage medium of claim 12,
2   wherein performing the interval equality constrained global optimization
3   process involves preconditioning the set of equality constraints through
4   multiplication by an approximate inverse matrix **B** to produce a set of
5   preconditioned equality constraints; and
6   wherein applying term consistency to the set of relations involves applying
7   term consistency to the set of preconditioned equality constraints over the subbox
8   **X**.

1   18.   The computer-readable storage medium of claim 17, wherein
2   applying box consistency to the set of relations involves applying box consistency
3   to the set of preconditioned equality constraints over the subbox **X**.

1   19.   The computer-readable storage medium of claim 12, wherein
2   performing the interval Newton step involves performing the interval Newton step
3   on the John conditions.

1   20.   The computer-readable storage medium of claim 12, wherein prior
2   to performing the interval Newton step on the John conditions, the method further
3   comprises performing a linearization test to determine whether to perform the
4   Newton step on the John conditions.

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1   21.   The computer-readable storage medium of claim 12, wherein

2   performing the interval equality constrained global optimization process involves:

3   evaluating a first termination condition;

4   wherein the first termination condition is TRUE if the width of the subbox

5   **X** is less than a pre-specified value, ε_X, and the width of the $f(\mathbf{X})$ is less than a pre-

6   specified value, ε_F; and

7   if the first termination condition is TRUE, terminating further splitting of

8   the subbox **X**.

1   22.   The computer-readable storage medium of claim 12, wherein

2   performing the interval Newton step involves:

3   computing $\mathbf{J}(\mathbf{x},\mathbf{X})$, wherein $\mathbf{J}(\mathbf{x},\mathbf{X})$ is the Jacobian of the function **f**

4   evaluated as a function of **x** over the subbox **X**; and

5   determining if $\mathbf{J}(\mathbf{x},\mathbf{X})$ is regular as a byproduct of solving for the subbox **Y**

6   that contains values of **y** that satisfy $\mathbf{M}(\mathbf{x},\mathbf{X})(\mathbf{y}\text{-}\mathbf{x}) = \mathbf{r}(\mathbf{x})$, where

7    $\mathbf{M}(\mathbf{x},\mathbf{X}) = \mathbf{B}\mathbf{J}(\mathbf{x},\mathbf{X})$, $\mathbf{r}(\mathbf{x}) = \text{-}\mathbf{B}\mathbf{f}(\mathbf{x})$, and **B** is an approximate inverse of the center of

8    $\mathbf{J}(\mathbf{x},\mathbf{X})$.

1   23.   An apparatus that solves a global optimization problem specified

2   by a function f and a set of equality constraints, the apparatus comprising:

3   a receiving mechanism that is configured to receive a representation of the

4   function f and the set of equality constraints $q_i(\mathbf{x}) = 0$ ($i=1,...,r$), wherein f is a

5   scalar function of a vector $\mathbf{x} = (x_1, x_2, x_3, ... x_n)$;

6   a memory for storing the representation;

7   an interval global optimization mechanism that is configured to perform

8   an interval equality constrained global optimization process to compute

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9   guaranteed bounds on a globally minimum value of the function $f(x)$ subject to the

10   set of equality constraints;

11   a term consistency mechanism within the interval global optimization

12   mechanism that is configured to apply term consistency to a set of relations

13   associated with the interval equality constrained global optimization problem over

14   a subbox X, and to exclude any portion of the subbox X that violates the set of

15   relations;

16   a box consistency mechanism within the interval global optimization

17   mechanism that is configured to apply box consistency to the set of relations

18   associated with the interval equality constrained global optimization problem over

19   the subbox X, and to exclude any portion of the subbox X that violates the set of

20   relations; and

21   an interval Newton mechanism within the interval global optimization

22   mechanism that is configured to perform an interval Newton step for the interval

23   equality constrained global optimization problem over the subbox X.

1   24.   The apparatus of claim 23, wherein the term consistency

2   mechanism is configured to apply term consistency to the set of equality

3   constraints $q_i(x) = 0$ ($i=1,...,r$) over the subbox X.

1   25.   The apparatus of claim 23, wherein the box consistency

2   mechanism is configured to apply box consistency to the set of equality

3   constraints $q_i(x) = 0$ ($i=1,...,r$) over the subbox X.

1   26.   The apparatus of claim 23,

2   wherein the interval global optimization mechanism is configured to,

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3   keep track of a least upper bound *f_bar* of the function $f(\mathbf{x})$,

4   and to

5   remove from consideration any subbox for which

6    $inf(f(\mathbf{X})) > f\_bar$;

7   wherein the term consistency mechanism is configured to apply term

8   consistency to the *f_bar* inequality $f(\mathbf{x}) \leq f\_bar$ over the subbox $\mathbf{X}$.

1   27.   The apparatus of claim 26, wherein the box consistency

2   mechanism is configured to apply box consistency to the *f_bar* inequality

3    $f(\mathbf{x}) \leq f\_bar$ over the subbox $\mathbf{X}$.

1   28.   The apparatus of claim 23,

2   wherein the interval global optimization mechanism is configured to

3   precondition the set of equality constraints through multiplication by an

4   approximate inverse matrix $\mathbf{B}$ to produce a set of preconditioned equality

5   constraints; and

6   wherein the term consistency mechanism is configured to apply term

7   consistency to the set of preconditioned equality constraints over the subbox $\mathbf{X}$.

1   29.   The apparatus of claim 28, wherein the box consistency

2   mechanism is configured to apply box consistency to the set of preconditioned

3   equality constraints over the subbox $\mathbf{X}$.

1   30.   The apparatus of claim 23, wherein the interval Newton

2   mechanism is configured to perform the interval Newton step on the John

3   conditions.

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1 31. The apparatus of claim 23, wherein prior to performing the interval

2 Newton step on the John conditions, the interval global optimization mechanism

3 is configured to perform a linearization test to determine whether to perform the

4 Newton step on the John conditions.

1 32. The apparatus of claim 23, wherein the interval global optimization

2 mechanism is configured to:

3   evaluate a first termination condition, wherein the first termination

4 condition is TRUE if the width of the subbox **X** is less than a pre-specified value,

5 ε_X, and the width of $f(\mathbf{X})$ is less than a pre-specified value, ε_F; and to

6   terminate further splitting of the subbox **X** if the first termination

7 condition is TRUE.

1 33. The apparatus of claim 23, wherein the interval Newton

2 mechanism is configured to:

3   compute **J(x,X)**, wherein **J(x,X)** is the Jacobian of the function **f** evaluated

4 as a function of **x** over the subbox **X**; and to

5   determine if **J(x,X)** is regular as a byproduct of solving for the subbox **Y**

6 that contains values of **y** that satisfy **M(x,X)(y-x)** = **r(x)**, where

7 **M(x,X)** = **BJ(x,X)**, **r(x)** = **-Bf(x)**, and **B** is an approximate inverse of the center of

8 **J(x,X)**.